

Study of Flavor Diagonal CP Violations on Lattice

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- R. Zhou, T. Blum, T. Doi, M. Hayakawa, TI, N. Yamada
(**R**iken-**B**NL-**C**olumbia collab.) [arXiv:0810.1302](#), [PRD76:114508](#).
- S. Aoki, R. Horsley, TI, Y. Nakamura, D. Pleiter, P. Rakow,
G. Schierholz, J. Zanotti [arXiv:0808.1428](#), [arXiv:0802.1470](#)



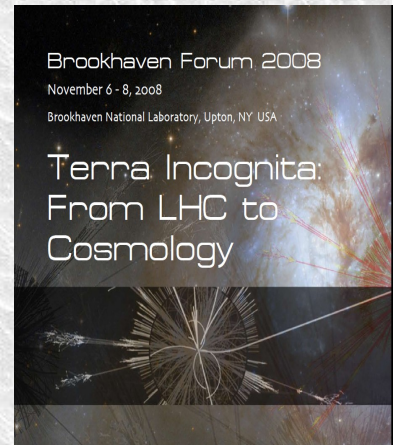
New Physics search on LHC and beyond

- **Baryon dominance** in the universe requires **extra** CP-violation (Sakharov criteria)
- Success of **Cabibbo-Kobayashi-Maskawa** theory, flavored CP violation is well described so far.
- Fine tuning in the **flavor diagonal CP violation**
 - Dimension 4: (the **Strong CP problem**)

$$\theta F^{\text{QCD}}_{\mu\nu} \tilde{F}^{\text{QCD}}_{\rho\lambda} = \theta F^{\text{QCD}}_{\mu\nu} F^{\text{QCD}}_{\rho\lambda} \epsilon_{\mu\nu\rho\lambda}$$

- Dimension 5: (can't be rotated away by axion)

$$\sum_{i=u,d,s,e,\mu} d_{i\text{QED}} \bar{\psi}_i F^{\text{QED}}_{\mu\nu} \sigma_{\mu\nu} \gamma_5 \psi_i + \sum_{q=u,d,s} d_{q\text{QCD}} \bar{q} F^{\text{QCD}}_{\mu\nu} \sigma_{\mu\nu} \gamma_5 q$$



EDM Experiments

- Neutron

$$|d_N| < 2.9 \times 10^{-26} \text{ e-cm}$$

Baker et al. (2007)

- ^{199}Hg

$$|d_{\text{Hg}}| < 2 \times 10^{-28} \text{ e-cm}$$

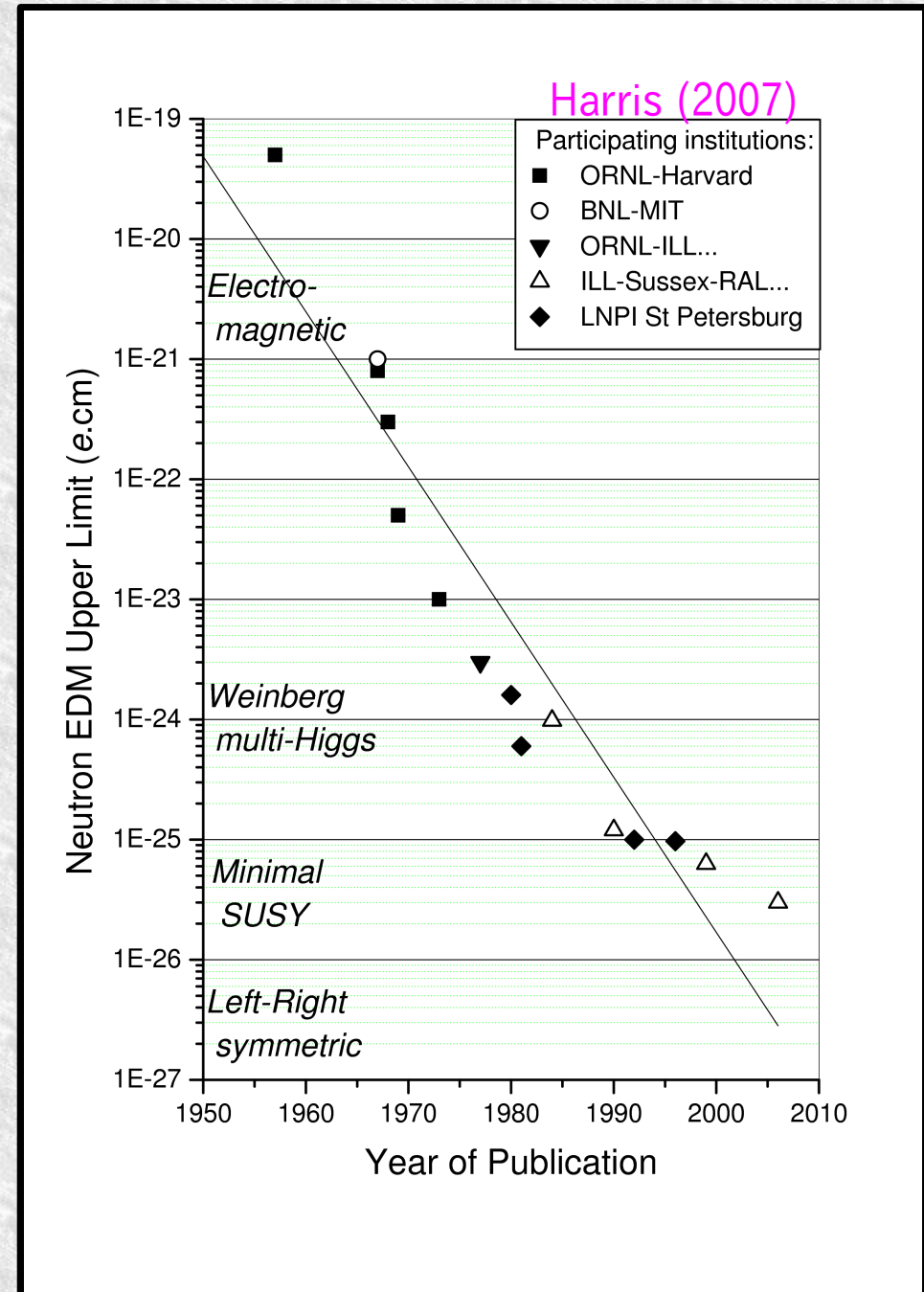
Romalis et al. (2000)

- ^{205}Tl

$$|d_{\text{Tl}}| < 9 \times 10^{-25} \text{ e-cm}$$

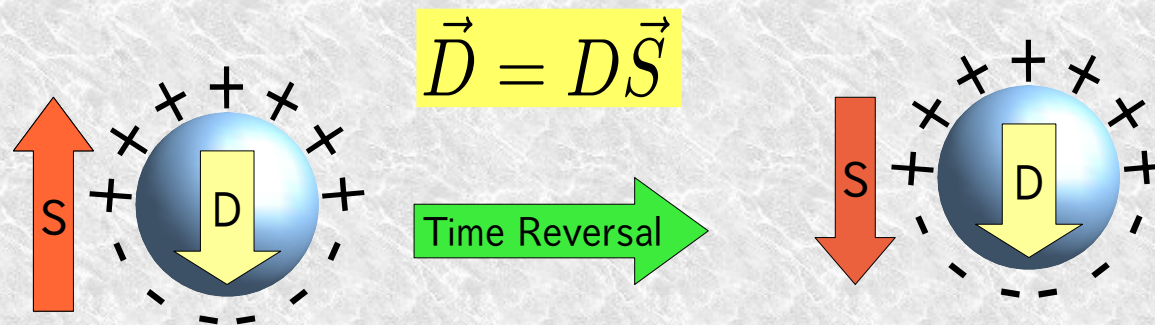
Regan et al. (2002)

- Many other plans including BNL Storage Ring EDM collaboration
deuteron EDM $\sim 10^{-29}$ e-cm



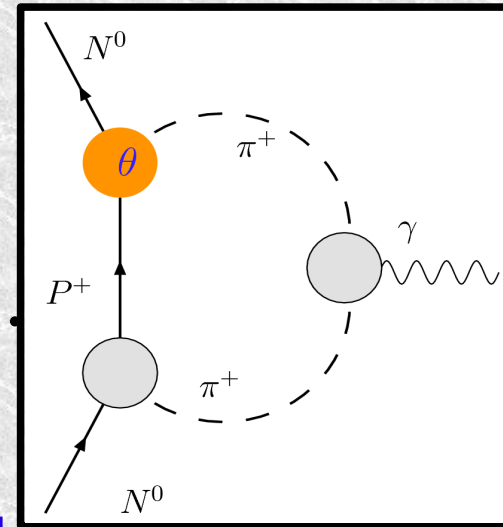
Electric Dipole Moment (EDM)

- Permanent **EDM** is a signature of **CP** (or Time reversal) symmetry **violation**.



- various candidates of **CP** violations :
 - **Electro Weak** (**CKM phase** in quark mass matrix) :
very small (Neutron: 10^{-4} smaller than exp. limit)
 - **New Physics** (**axion**, **SUSY**, **left-right**, **multi Higgs**)
 - **Strong CP**: vacuum angle θ This talk

Strong CP problem



$$S_\theta = i\theta \frac{1}{32\pi^2} \int d^4x F_{\mu\nu}(x) \tilde{F}_{\mu\nu}(x) = i\theta Q_{\text{top}}$$

Q_{top} is (classically) integer counting **winding number** of gluon field (net number of **instanton**). θ is an angle of complex phase assoc. each topological sectors: θ vacuum

- Neutron's Electric Dipole Moment, **NEDM** (**CP odd**) is experimentally very small,

$$D_n \leq 2.9 \times 10^{-13} [\text{e fm}] \quad (07, \text{Grenoble grp.})$$

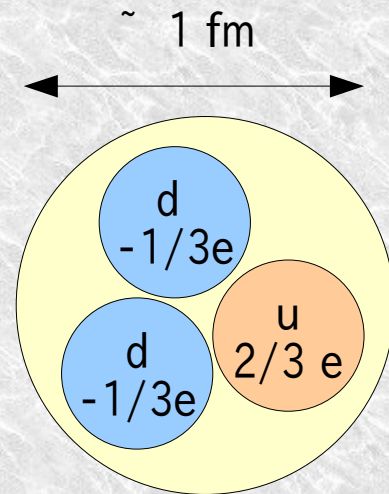
- Model estimates (Bag model, ChPT, Large N_c)

$$D_n = (9, 17, -4 \sim -7) \times 10^{-3} \times \theta [\text{e fm}]$$

(79 Baluni, 79 Crewther et.al., 92 Aoki-Hatsuda, 02 Pospelov-Ritz)

naturalness

$$|\bar{\theta}| = |\theta + \theta_{\text{EW}}| \leq 10^{-10}$$



Proposed solutions

- theta term is U(1) **ABJ Anomaly**:
changes the **path integral measure**

$$\begin{aligned}\psi &\rightarrow e^{i\theta'\gamma_5}\psi, \\ \bar{\psi} &\rightarrow \bar{\psi}e^{i\theta'\gamma_5}\end{aligned}$$

$$\mathcal{D}\psi\mathcal{D}\bar{\psi} \rightarrow \mathcal{D}\psi\mathcal{D}\bar{\psi} e^{i\frac{N_f}{32\pi^2}\theta' F_{\mu\nu}\tilde{F}_{\mu\nu}}$$

and also **quark mass terms**

$$m\bar{\psi}\psi \rightarrow m\bar{\psi}\psi \cos(\theta') + im\bar{\psi}\gamma_5\psi \sin(\theta')$$

- if up(/down) **quark is massless** (m=0) then vacuum angle can be rotated away, which is **less consistent** with spectrum results as we will see in the next pages. (Creutz Ann. Phys.322:1518,2007)
- A very light, neutral, weakly coupled pseudoscalar, **axion**, which is (would-be) NG-boson of the **Peccei-Quinn**

Symmetry

$$\left(\theta - \frac{a(x)}{f_A}\right) F_{\mu\nu}\tilde{F}_{\mu\nu}$$

QCD+QED simulation

- The first principle calculations of **isospin breaking** effects due to **electromagnetic(EM)** and the up, down **quark mass difference** are necessary for accurate determination of quark masses.

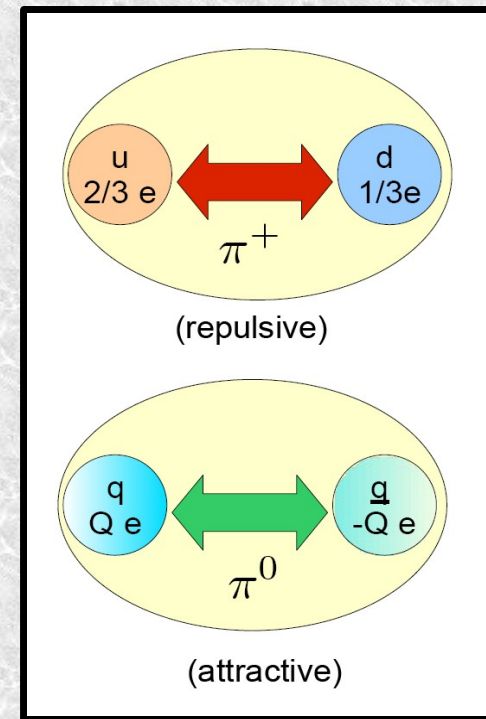
- EM splittings are measured very accurately:

$$m_{\pi^{\pm}} - m_{\pi^0} = 4.5936(5) \text{ MeV}$$

$$m_N - m_P = 1.2933317(5) \text{ MeV}$$

- $O(\alpha)$ radiative corrections in Hadron structure is **a major uncertainties** e.g.

$$f_{\pi^{\pm}} = 130.7 \pm 0.1 \pm \mathbf{0.36} \text{ MeV} \quad \text{PDG}$$



EM splittings

- ChPT with EM at $O(p^4, p^2 e^2)$:

Urech, NPB433 (95) 234 Bijnens et al. PRD (07) 014505

$$M_{\pi^\pm}^2 = 2mB_0 + 2e^2 \frac{C}{f_0^2} + \mathcal{O}(m^2 \log m, m^2) + I_0 e^2 m \log m + K_0 e^2 m$$

$$M_{\pi^0}^2 = 2mB_0 + \mathcal{O}(m^2 \log m, m^2) + I_\pm e^2 m \log m + K_\pm e^2 m$$

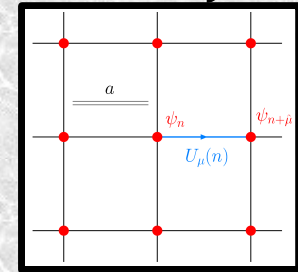
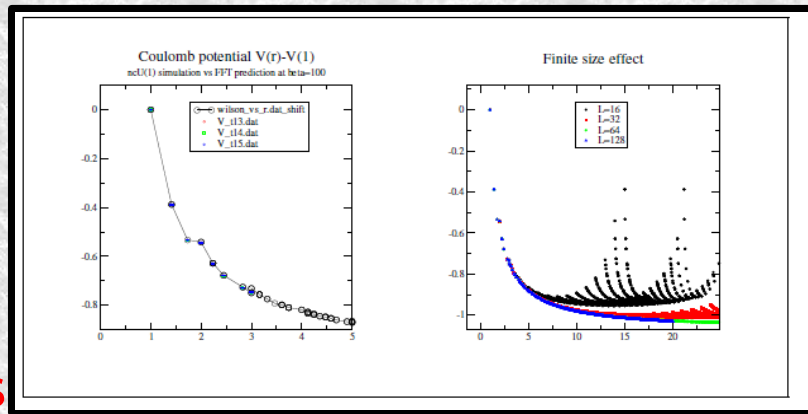
- Neutral pion, kaons are still **Nambu-Goldstone boson** @ $m_q=0$
- Dashen's theorem** : The difference of squared pion mass is independent of quark mass upto $O(e^2 m)$

$$\Delta M_\pi^\gamma \equiv M_{\pi^\pm}^\gamma - M_{\pi^0}^\gamma = \gamma e^\gamma \frac{C}{f_\gamma} + (I_\pm - I_\gamma) e^\gamma m \log m + (K_\pm - K_\gamma) e^\gamma m$$

- C, K_\pm, K_0 are **new low energy constants** determined on **QED+QCD simulation** (I_\pm, I_0 are known functions).

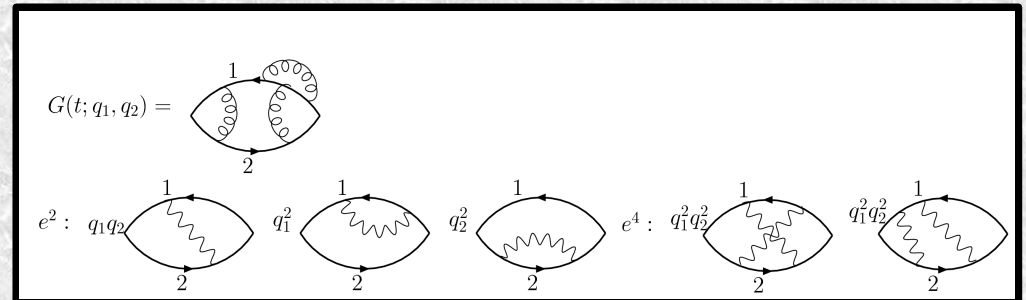
EM splittings on lattice

- Full QCD: **Nf=2** (& 2+1) **dynamical DWF quarks**
- Free photon field (non-compact lattice U(1) gauge field) interacts only with valence quark (dynamical QCD but quenched QED)
- Lattice spacings $a^{-1} = 1.69(53)$ [GeV] or $a \sim 0.12$ [fm]
Volume $\sim (1.9 \text{ fm})^3$
- About **200 x 2 statistical samples** of **QCD x QED** vacuum for each quark mass points (50, 75, 100 % of strange quark mass) are used to get the Hadron propagators .



$$C_X(t) = A(e^2)e^{-M_X(e^2)t}$$

$$\frac{C_{X^\pm}(t) - C_{X^0}(t)}{C_{X^0}(t)} = \Delta M_X \times t + \text{Const.}$$



EM splittings results

- Fit 61 x 3 PS masses to get EM-LECs

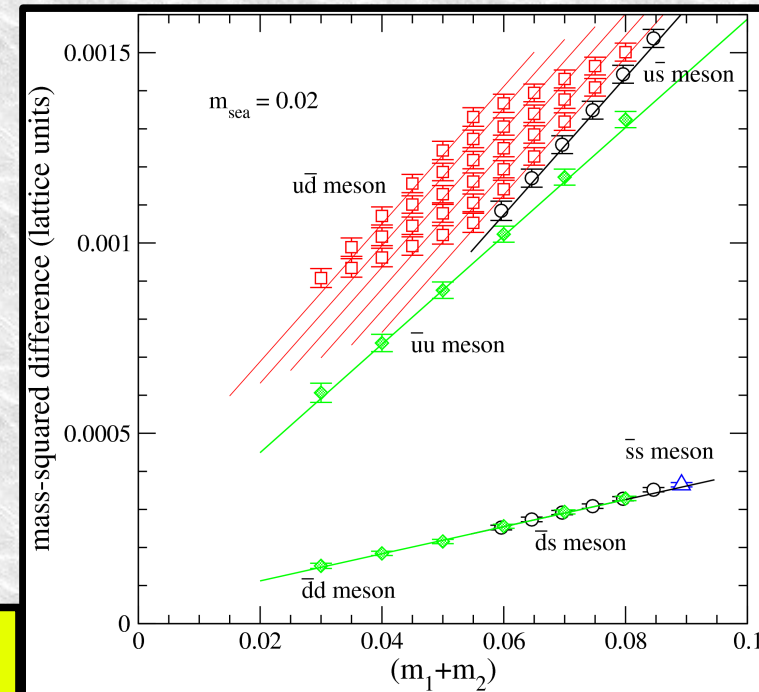
- Input experimental PS mesons

$$m_{\pi^\pm}^2, m_{K^\pm}^2, m_{K^0}^2 \quad (\text{no } m_{\pi^0}^2)$$

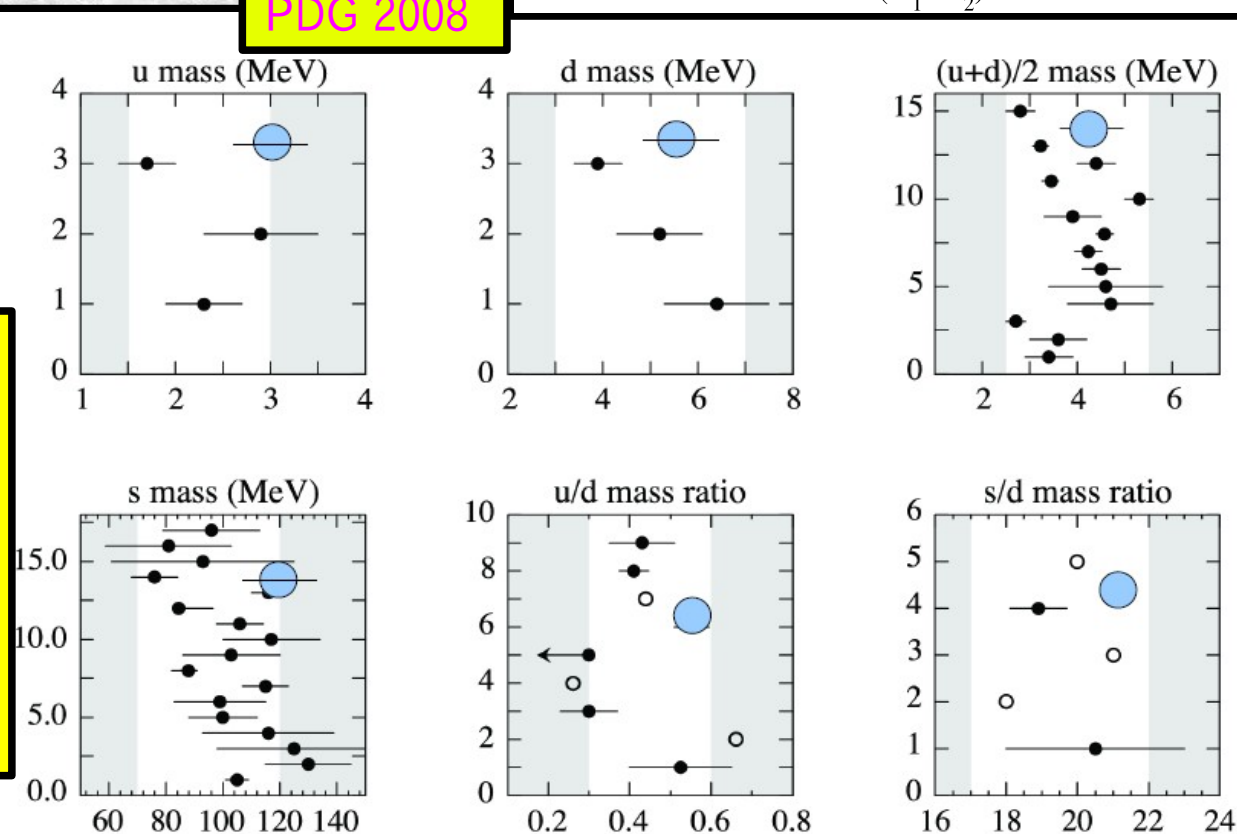
- Non-perturbative renormalization

- Massless quarks are unlikely consistent with experiments

Quoted in
PDG 2008



$$\begin{aligned} m_u^{\overline{\text{MS}}}(2 \text{ GeV}) &= 3.02(27)(19) \text{ MeV}, \\ m_d^{\overline{\text{MS}}}(2 \text{ GeV}) &= 5.49(20)(34) \text{ MeV}, \\ m_{ud}^{\overline{\text{MS}}}(2 \text{ GeV}) &= 4.25(23)(26) \text{ MeV}, \\ m_s^{\overline{\text{MS}}}(2 \text{ GeV}) &= 119.5(56)(74) \text{ MeV}, \\ m_u/m_d &= 0.550(31), \\ m_s/m_{ud} &= 28.10(38). \end{aligned}$$



Nucleon EDM on Lattice

- 18 years of Nucleon Electric Dipole Moment, NEDM on lattice : **two methods** so far

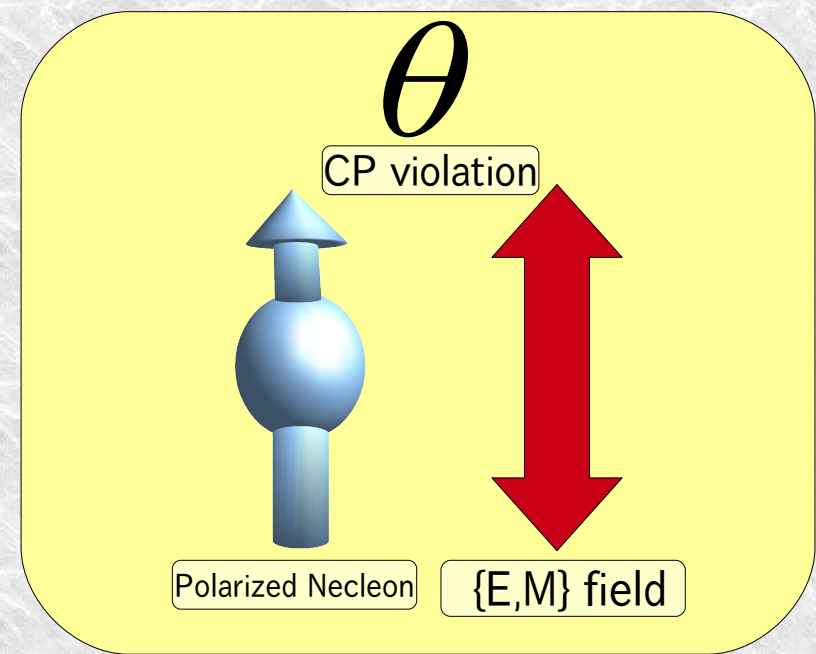
89 Aoki-Gocksch	Nf=0 Wilson	Electric field
04 Berruto et. al.	Nf=0 DWF	3pt
05 Berruto et. al.	Nf=2 DWF	3pt
06 Shintani et. al. (CP-PACS)	Nf=0 DWF	3pt
06 Shintani et. al. (CP-PACS)	Nf=0 DWF, Nf=2 clover	Electric field
06 QCDSF	Nf=0 overlap	3pt
06 Blum, TI, & Doi	Nf=2 DWF	Electric field
06 Shintani et. al. (CP-PACS)	Nf=2 clover	Electric field

(c.f. **experiments: 10^{-6} progress for last ~50 years**)

- **Dynamical Simulation** (including **sea quark** pair creation/annihilation effect) becomes easier. NEDM is very sensitive to **sea quark mass**; zero at chiral limit (and valence quark theta is to be rotated away)

NEDM simulation

- Three ingredients
 - Uniform Electric Field by a boundary condition
(98 U. Heller, 64 E. Brown)
 - source of CP violation θ
 - Polarized Nucleon
- Measure a spin splitting of Nucleon's energy
(89 Aoki-Gocksch)



$$E = 2\pi/(N_t N_z) \Rightarrow e^{-i 15 E} = e^{i E}$$

1	$e^{i E}$	$e^{i 2E}$	$e^{i 3E}$	$e^{-i 12 E}$
1	$e^{i E}$	$e^{i 2E}$	$e^{i 3E}$	$e^{-i 8 E}$
1	$e^{i E}$	$e^{i 2E}$	$e^{i 3E}$	$e^{-i 4 E}$
1	$e^{i E}$	$e^{i 2E}$	$e^{i 3E}$	1
	(N _t =N _z =4)			T

$$m_{N\theta}(E, \uparrow) - m_{N\theta}(E, \downarrow) = \imath i \theta d_N \mathbf{S} \cdot \mathbf{E} + \mathcal{O}(E^2, \theta^2)$$

Dynamical QCD with θ

- Source of CP violation : $S_\theta = i \frac{\theta}{\imath \imath \pi \imath} \sum \tilde{F} F = i\theta Q_{\text{top}}$
- In previous calculations, $\theta = 0$ ensemble were generated (Nf=0,2), then each configuration were reweighted by topological charge : $\langle \mathcal{O} \rangle_\theta = \langle \mathcal{O} e^{i\theta Q} \rangle$.
- Alternatively, one could put θ term into the ensemble probability, rather than observable, if θ is **analytically continued to pure imaginary**.
- Spin splitting becomes real:

$$m_{N\theta}(E, \uparrow) - m_{N\theta}(E, \downarrow) = \imath i\theta d_N S \cdot E \longrightarrow -\imath \theta d_N S \cdot E$$

$$\theta \longrightarrow -i\theta$$

previously done by Minkowskian E . .

85 Bhanot-David, 02 Azcoiti et. al,
06 Imachi-Yoneyama et.al.
(CP^N model with imaginary theta)

02 de Forcrand-Philipsen,
03 D'Elia-Lombardo (chem. pot.) 13

Simulation with imaginary θ term

- By using **anomalous chiral WT** theta term is absorbed into CP-odd “**quark mass term**”.

$$\begin{aligned}\bar{\psi} &\longrightarrow \bar{\psi} e^{\theta \gamma_5} \\ \psi &\longrightarrow e^{\theta \gamma_5} \psi\end{aligned}$$

$$\mathcal{D}\psi\mathcal{D}\bar{\psi} e^{-S_\theta} \rightarrow \mathcal{D}\psi\mathcal{D}\bar{\psi}$$

(flavor-singlet axial “U(1)” rotation)

$$\mathcal{L}_\theta = \frac{m\theta}{2} \bar{\psi} \gamma_5 \psi$$

- Nf=2** clover fermion
- V ~ (2 fm)³** (16³ x 32)
- a⁻¹ = 2 GeV** (beta=2.1 Iwasaki)
- Quark mass: **mPS/mV = 0.85**
- $\theta = 0.4, 0.0, (0.2)$**
- ~ 5,000/50 vacuum samples** for each parameter set

$$\psi(D_W + m + ia \frac{c_{SW}}{\xi} \sigma_{\mu\nu} F_{\mu\nu})\psi(x)$$

$$F_{\mu\nu}^C = \frac{1}{\xi} \text{An} \left(\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \right), \quad F_{\mu\nu}^R = \frac{1}{\xi} \text{An} \left(\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} + \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \right),$$

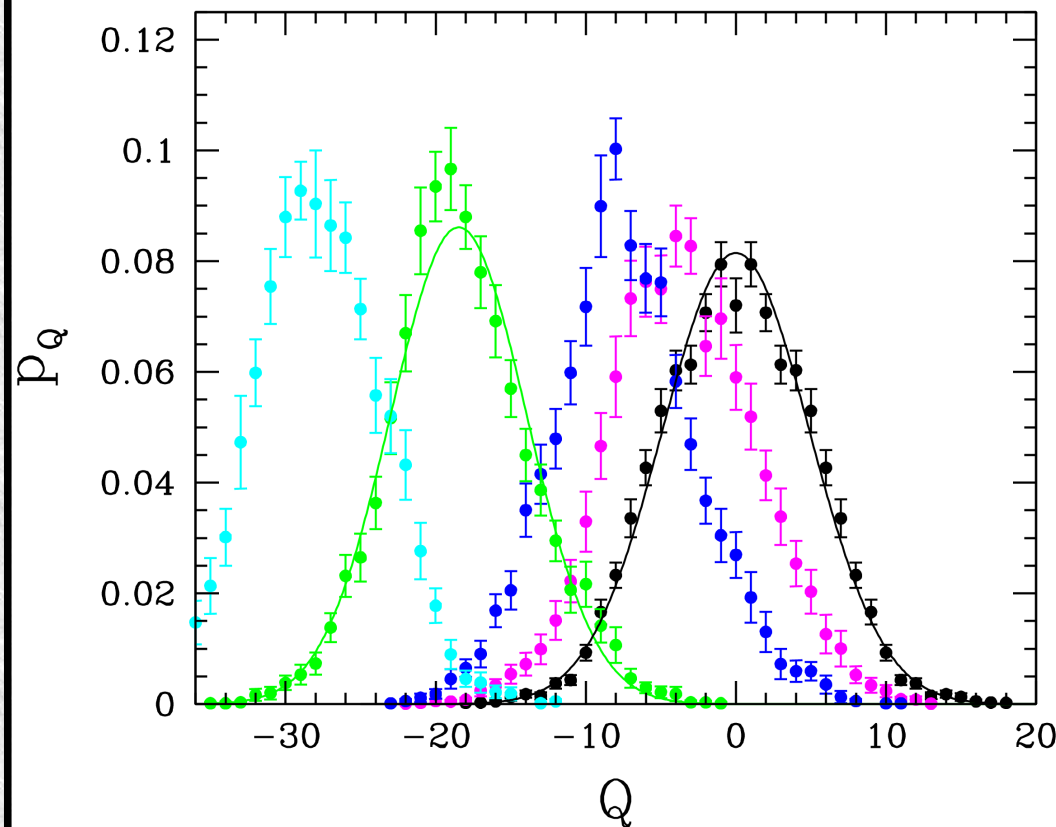
$$\theta'' = \left[1 - \frac{\kappa}{\kappa_c(\beta)} \right] Z_P \theta$$

Topological charge distribution

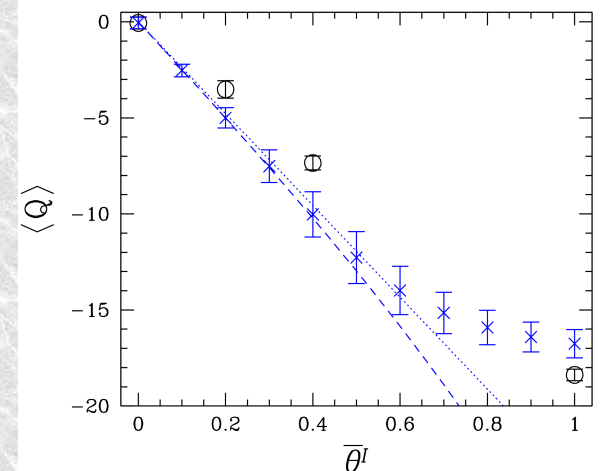
$$Q_{\text{top}} = \frac{1}{32\pi^2} \sum \tilde{F}F(x)$$

$$F_{\mu\nu}^C = \frac{1}{4} \text{An} \left(\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} \right), \quad F_{\mu\nu}^R = \frac{1}{16} \text{An} \left(\begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} + \begin{array}{|c|c|} \hline \square & \square \\ \hline \hline \end{array} \right),$$

$$\theta = 1.5, 1.0, 0.4, 0.2, 0.0$$



- Topological charge Q_{top} from gluon field (APE cooling)
- CP symmetry is successfully broken by the imaginary θ .
- Roughly consistent with the reweighting data

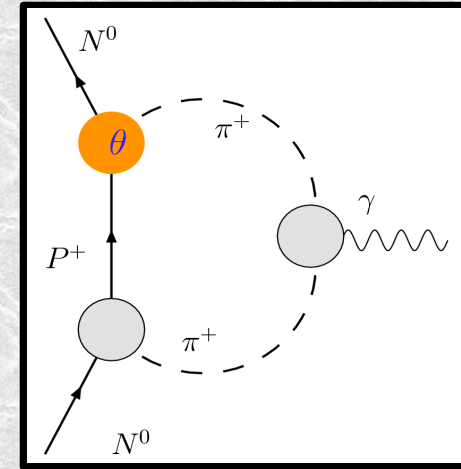


Preliminary results of NEDM

- The **third** smallest Uniform Electric field

$$E_Z = \frac{2\pi}{N_z N_t} \times 3 \simeq 0.0368$$

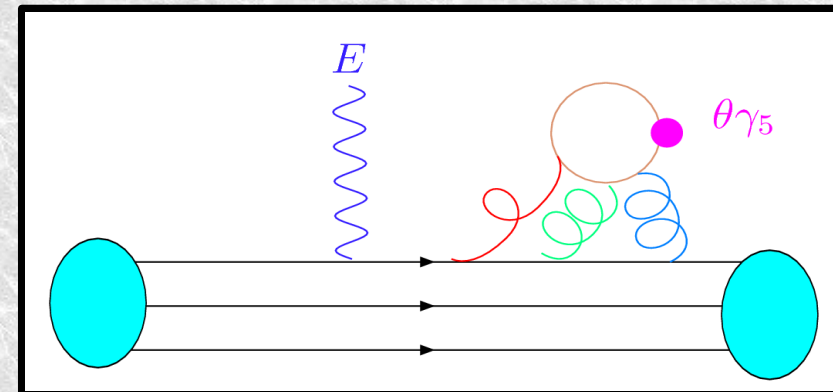
- quark EM charges = $\{+, -\}$ 1, 2 $\times \{2/3, -1/3\}$
- tsrc = 0, 16, Nucleon 2pt corr. func. (4x4)



$$\frac{\langle N_{\uparrow}(t) \bar{N}_{\uparrow}(t_{src}) \rangle}{\langle N_{\downarrow}(t) \bar{N}_{\downarrow}(t_{src}) \rangle} \rightarrow c e^{\Delta m t}$$

$$\Delta m = m_{N\theta}(E, \uparrow) - m_{N\theta}(E, \downarrow)$$

$$= -2\theta d_N \mathbf{S} \cdot \mathbf{E}$$

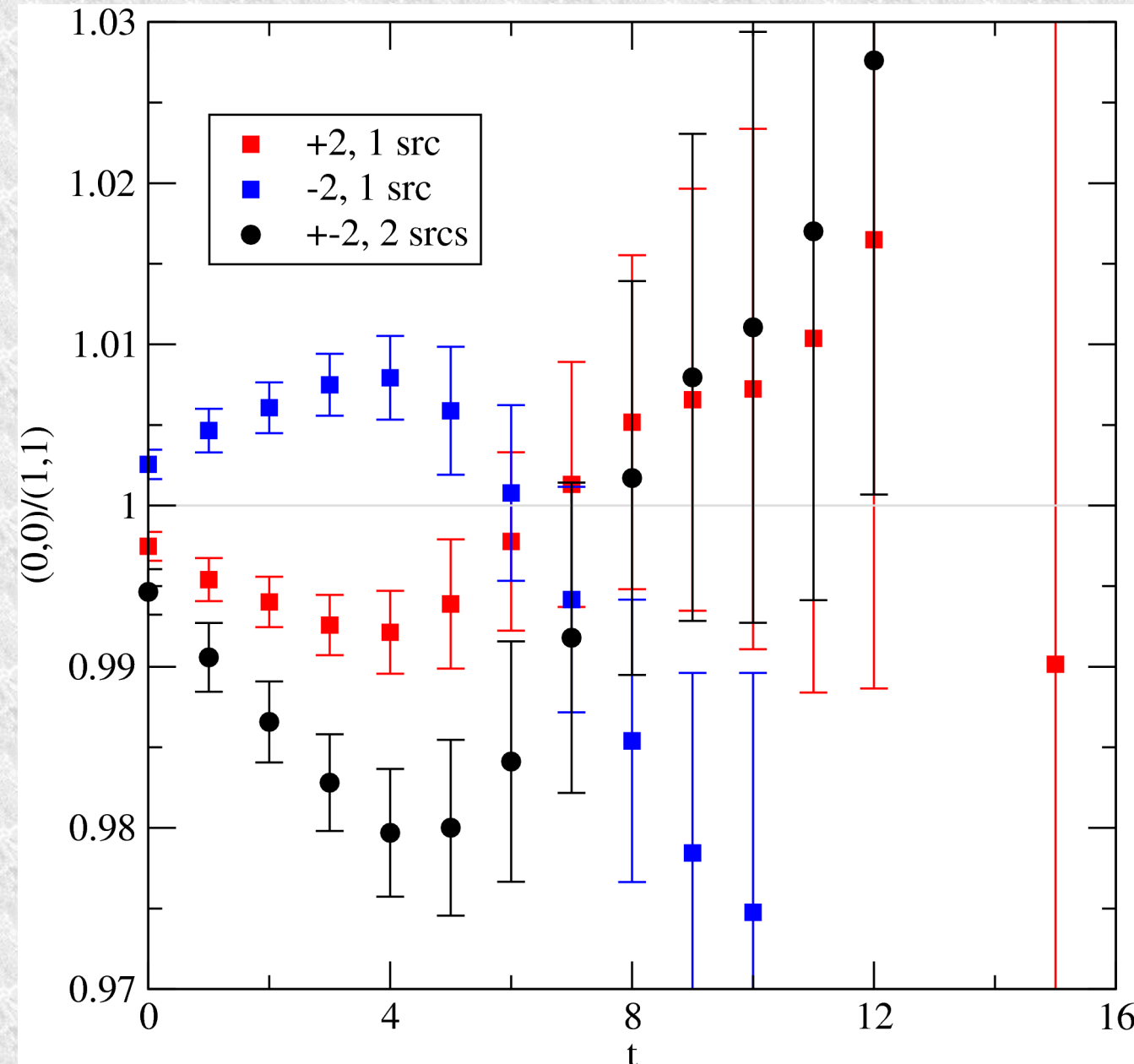


- Spin splittings of the Nucleon propagator

$$\frac{\langle N_{\uparrow}(t) \bar{N}_{\uparrow}(t_{src}) \rangle}{\langle N_{\downarrow}(t) \bar{N}_{\downarrow}(t_{src}) \rangle} \rightarrow c \exp(\Delta m t) \simeq c(1 + \Delta m t)$$

- Signal gain by
 - taking ratio of the propagators
 - averaging $\pm E$, cancels $O(E^0)$
 - many sources

signal at $t < 5$ excited state contamination needs to be checked

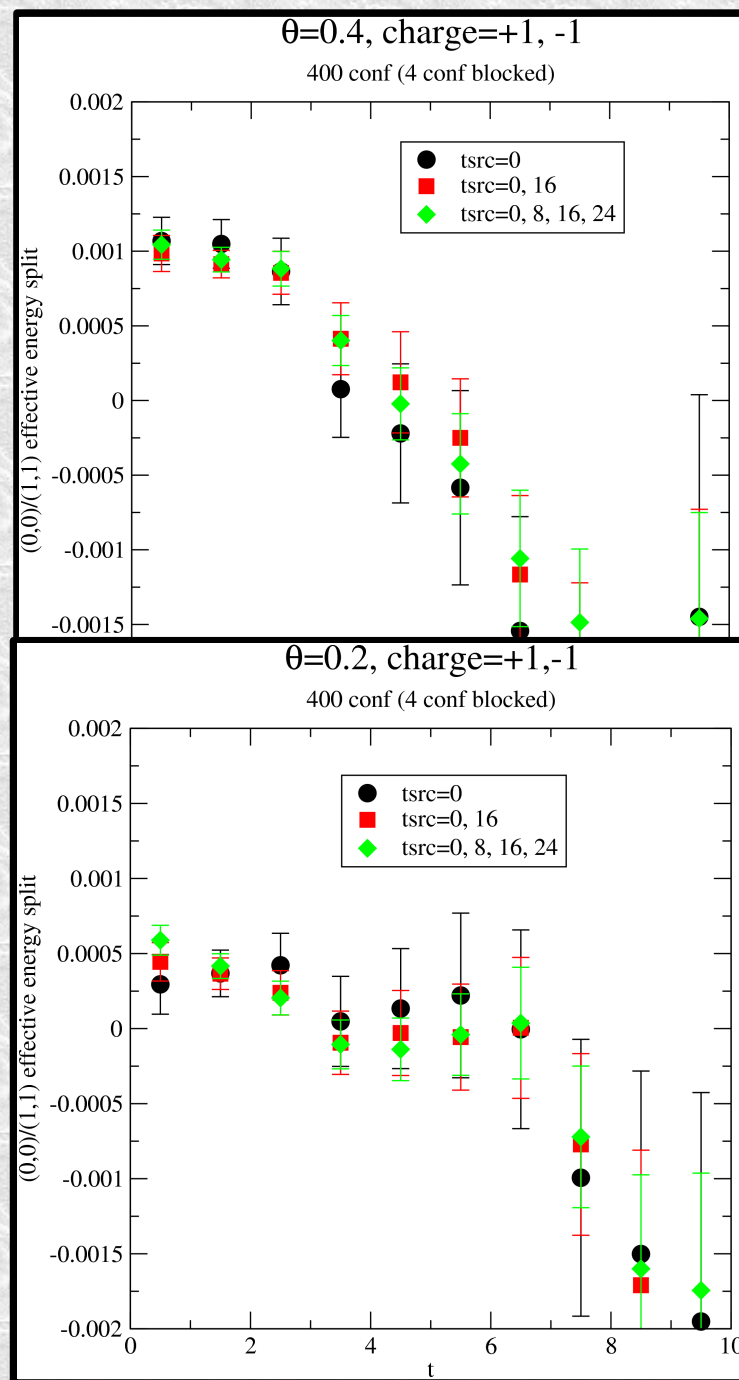


effective energy splitting

$$\Delta m = m_{N\theta}(E, \uparrow) - m_{N\theta}(E, \downarrow)$$

$$= -2\theta d_N S \cdot E$$

- average t , $Nt-t$ at propagator level
- numbers are comparable to the previous estimations
- scaling with θ indicates physical signal (no correlation)

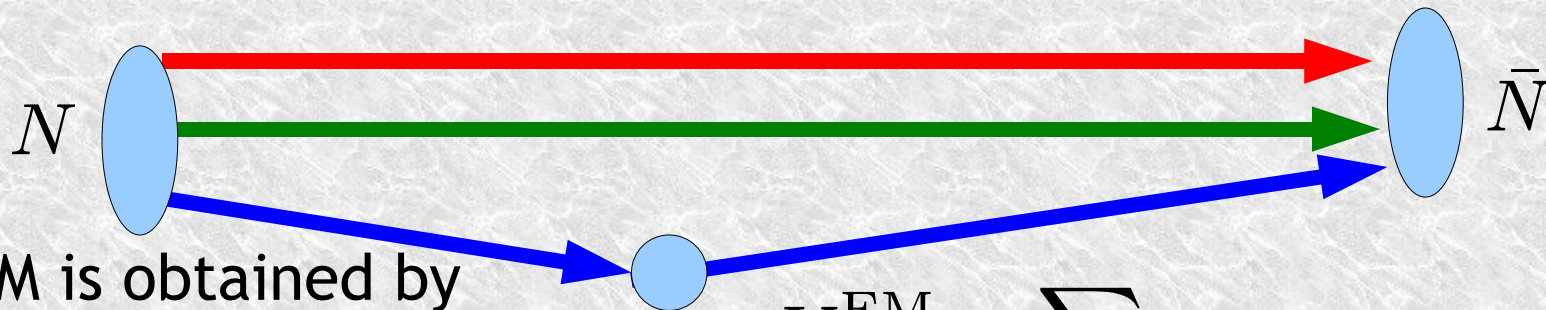


EDM from form factors (3pt function)

- Electric form factors F3

(04 Berruto, Blum, Orginos, Soni, 05 Shintani et.al CP-PACS)

$$\left\langle N_s(\mathbf{p}') | V_\mu^{EM}(\mathbf{q}) | \bar{N}_s(\mathbf{p}) \right\rangle_\theta = F_1(q^2) \gamma_\mu + F_2(q^2) \frac{q_\nu \sigma_{\mu\nu}}{2m_N} + i\theta F_3(q^2) \frac{q_\nu \sigma_{\mu\nu} \gamma_5}{2m_N} + \dots, \quad q = p' - p$$



- EDM is obtained by

$$V_\mu^{EM} = \sum_q e_q \bar{q} \gamma_\mu q$$

$$D_n = \lim_{q^2 \rightarrow 0} \frac{e}{2m_N} F_3(q^2)$$

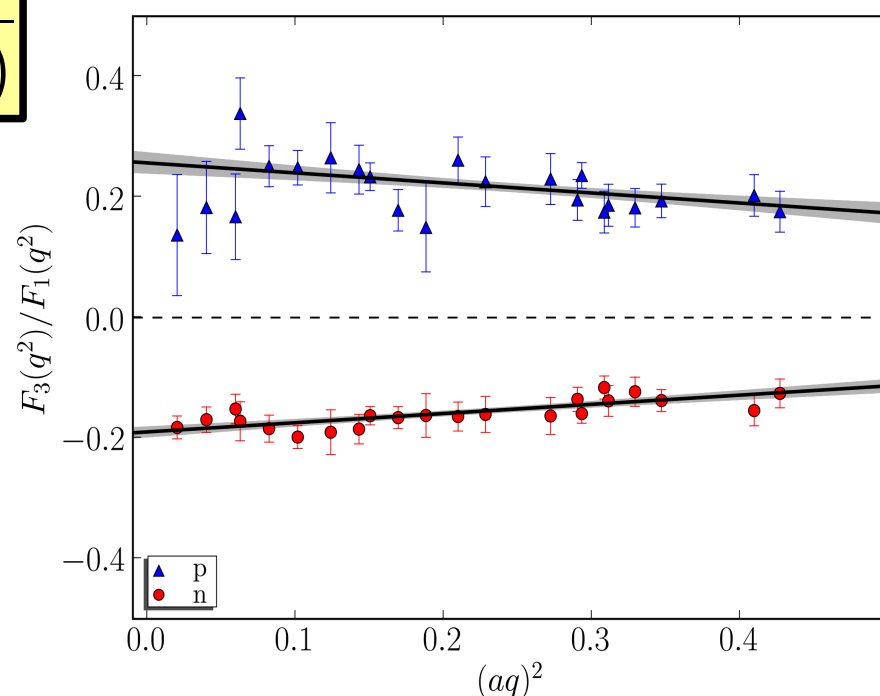
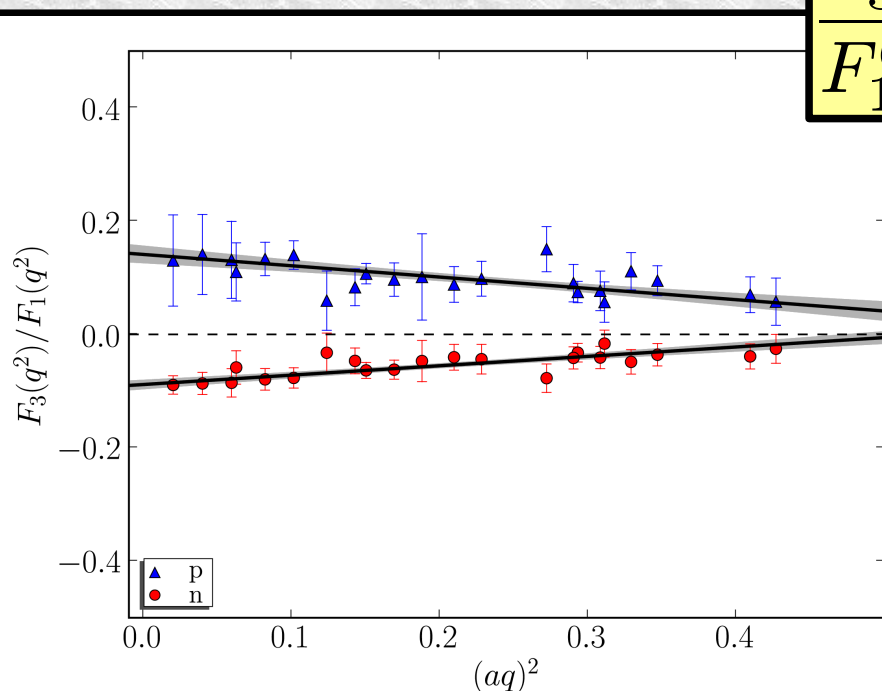
Preliminary results of F_3

- valence theta = sea theta = 0.2 (left) and 0.4 (right)

- Dipole ansatz

$$F_3^\theta(q^2) = \frac{F_3^\theta(0)}{(1 + q^2/M^2)^2}$$

$$\frac{F_3^\theta(q^2)}{F_1^\theta(q^2)}$$



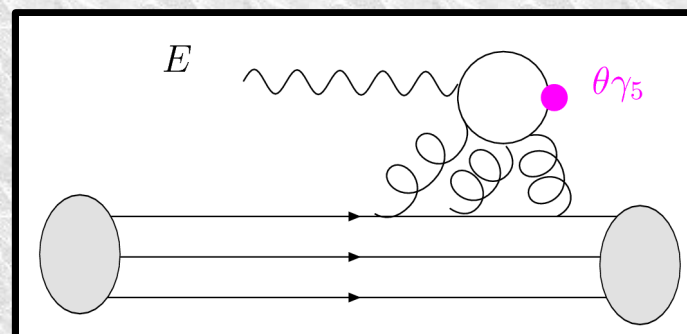
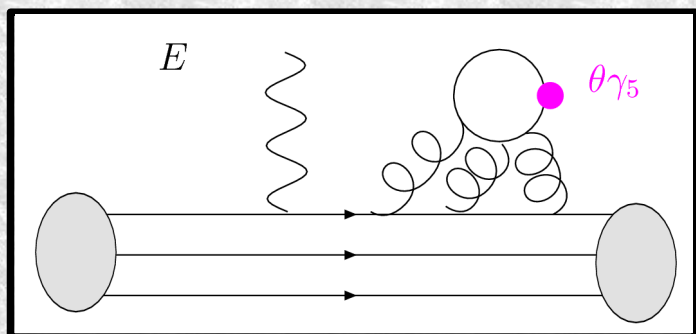
Summary & Discussions

- QCD+QED DWF simulation is carried out to determine quark masses: massless quarks are unlikely consistent with experimental spectrum so far(DWF preserves chiral symmetry).
- More realistic calculation on $N_f=2+1$ is being carried out.
- Dynamical clover simulation with imaginary θ term could control topology distribution of QCD ensemble.
- Both the external field method and the form factor calculation show signals for relatively heavy quark mass.
- (re)normalization of θ is needed along with the usual mass term, at least, on lattice (for clover quark, chiral symmetry is broken).
- More careful studies are needed for physical results.

Future plan

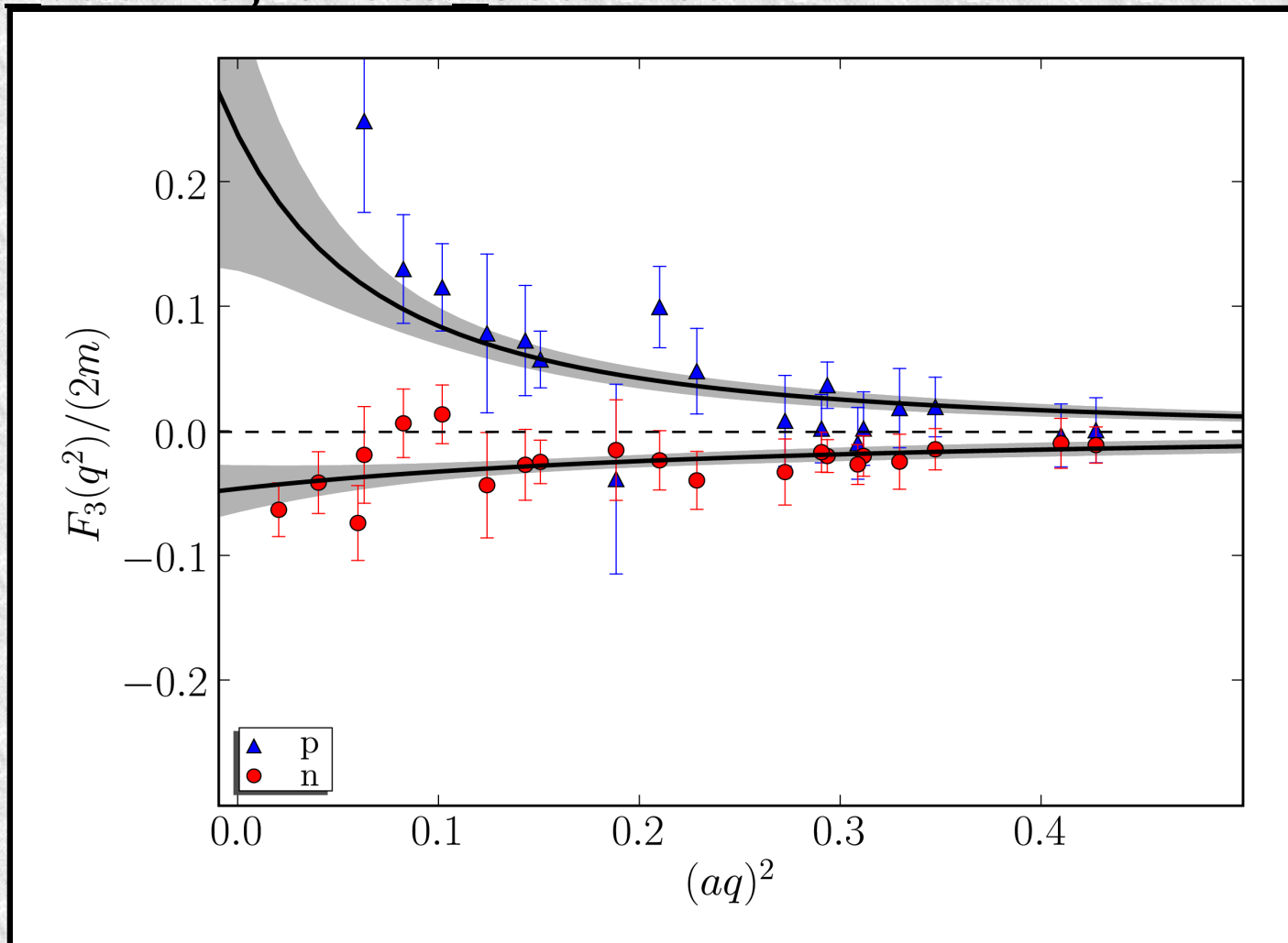
- Improving current calculation includes
 - More statistics
 - several θ values to check $\mathcal{O}(\theta^2)$
 - quark mass/ volume/ lattice spacing dependences
 - **Valence θ** dependence [Aoki-Gocksh-Manohar-Sharpe 90]
- NEDM needs another disconnected quark loop contribution.

(07 Shintani et.al.)



Dependence to the valence theta

- [Aoki-Gocksh-Manohar-Sharpe 90] argued **valence theta dependence** is **unphysical lattice artifact** and shall be rotated away by U(1) **chiral transformation**. The clover fermion explicitly breaks chiral symmetry as a lattice artifact (c.f. DWF).
- $\theta_{\text{val}} = 0, \theta_{\text{sea}} = 0.4$

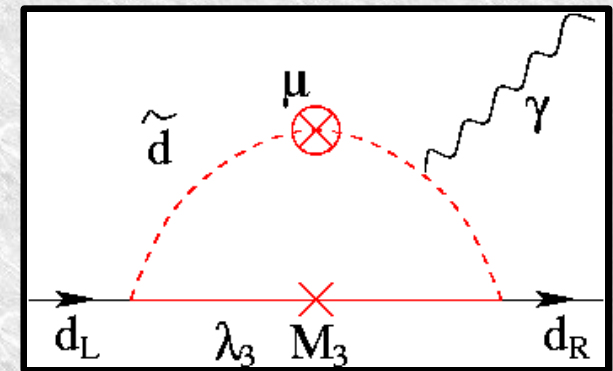


Future prospects

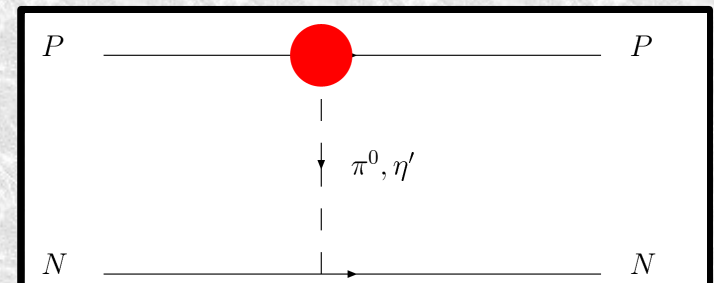
- Effects of **Quark EDM** on lattice ?

('03 Hisano-Shimizu, Nagai et. al.)

- $i \frac{d_q^C}{2} \bar{\psi} \sigma_{\mu\nu} \gamma_5 F_{\mu\nu} \psi$ may be doable by $d_q^C \rightarrow -i d_q^C$
- Has mixing with other operators



- Study of other Hadrons Deuteron, ... , Hg, Tl's **EDM** ?
measuring CP-violating pi-N-N effective coupling

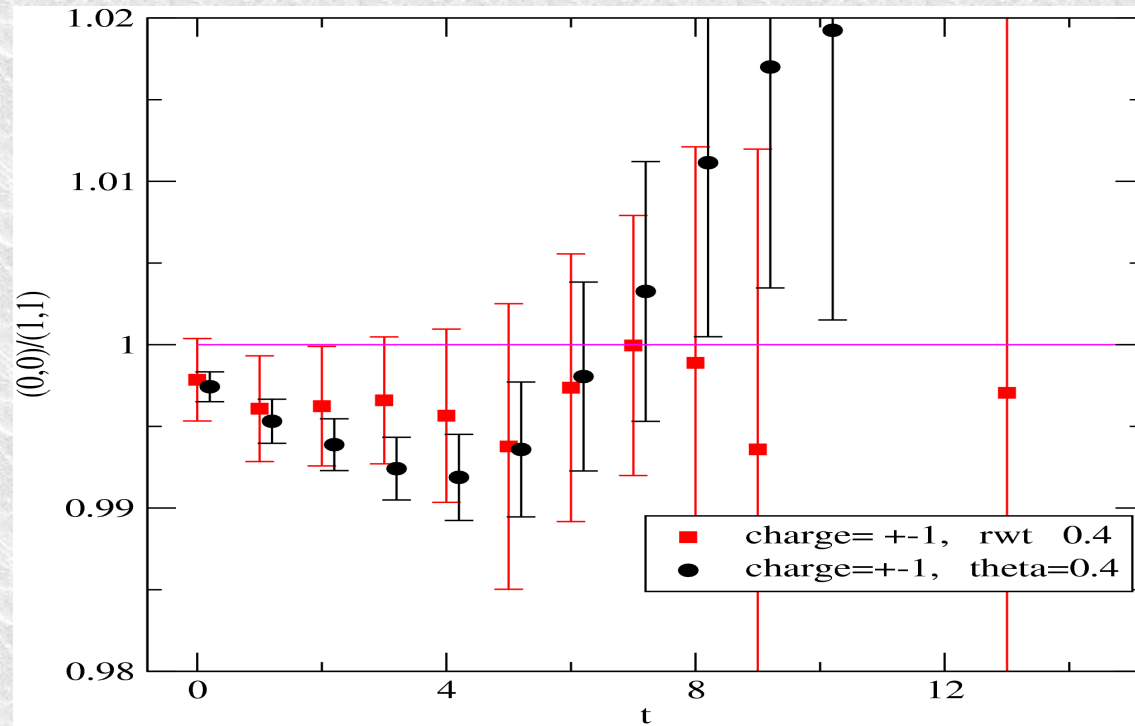


Acknowledgments

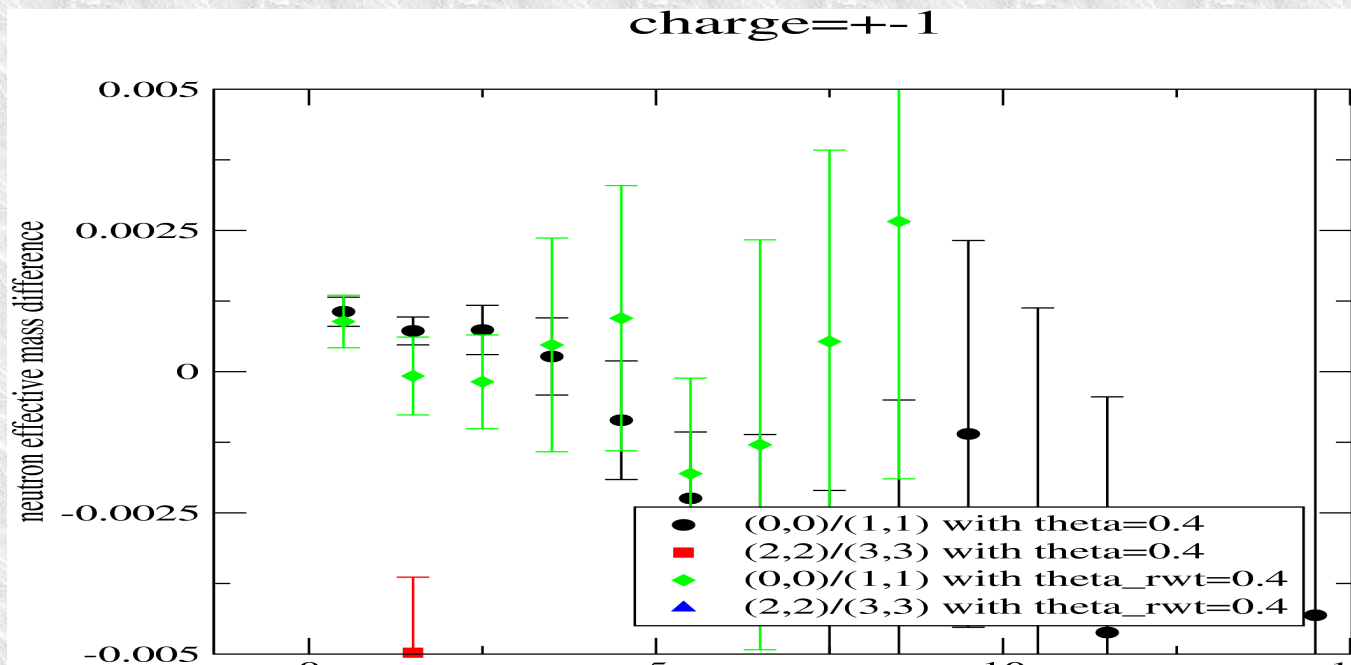
- $N_f=2$ & $2+1$ DWF is generated by the RBC-UKQCD collaboration.
- We are grateful for authors/maintainers of DESY code and CPS++, which are used for ensemble generation and measurements.
- We thank for computational resources supported by the Large Scale Simulation Program No. 07-14 (FY2007) of High Energy Accelerator Research Organization (KEK), the RIKEN Super Combined Cluster (RSCC), and QCDOC at RBRC & Columbia University.
- This work is supported in part by Grants-in-Aid for Scientific Research from the Ministry of Education, Culter, Sports, Science and Technology, No. 17750050 (FY2007-2008), and JSPS & DFG Japan-German Corporative Research Program (FY2008-2009).

Appendix

comparison with reweighting method



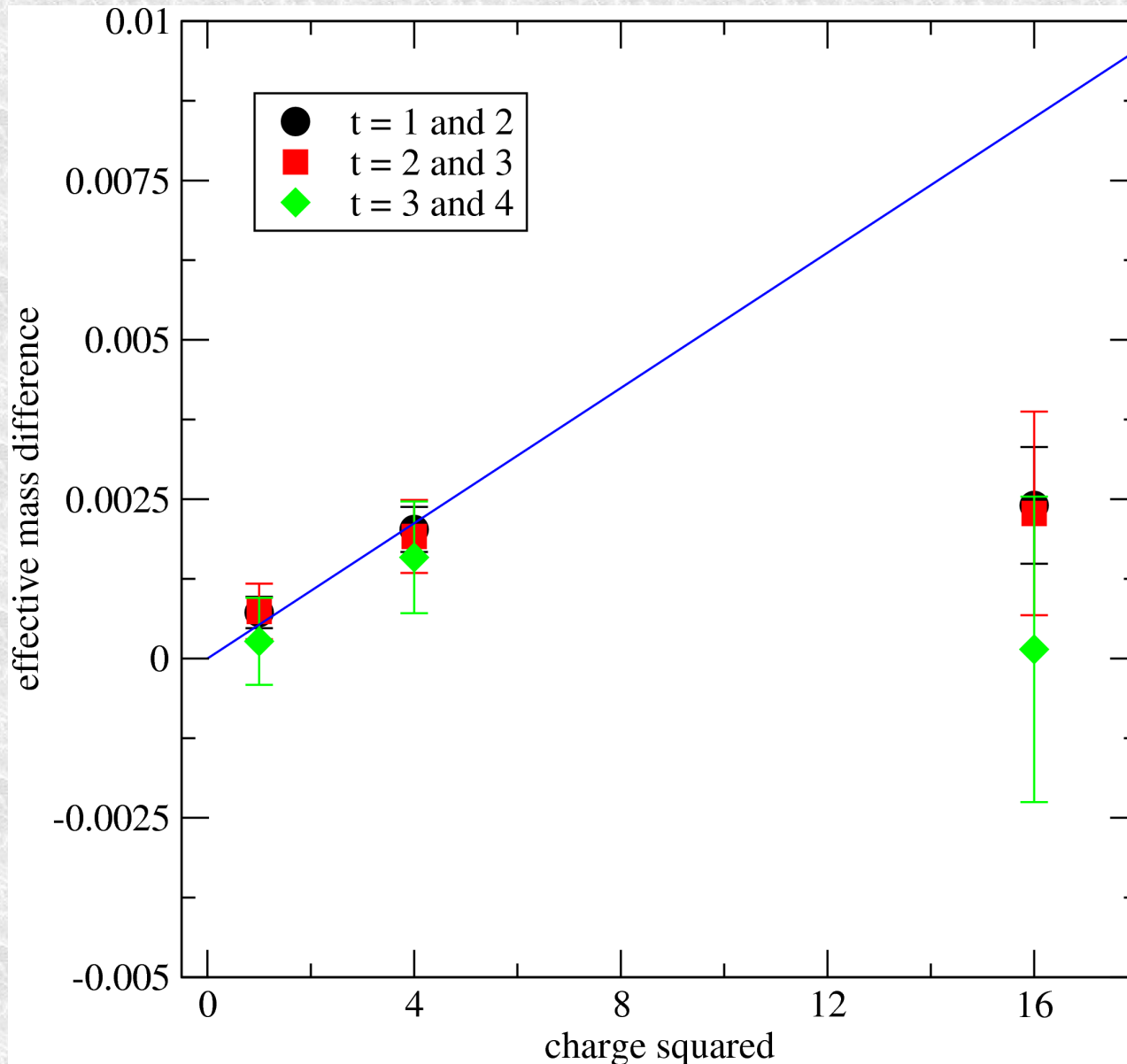
- charge = +/- 1
- reweighting method has larger error bar



- Further nested ratio ?

- $G(\theta = 0)$
- $G(\text{mag})$

Electric charge dependence

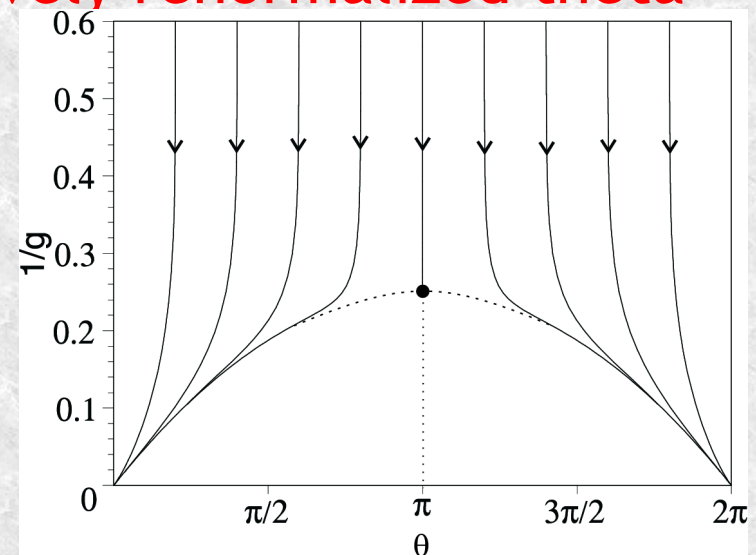
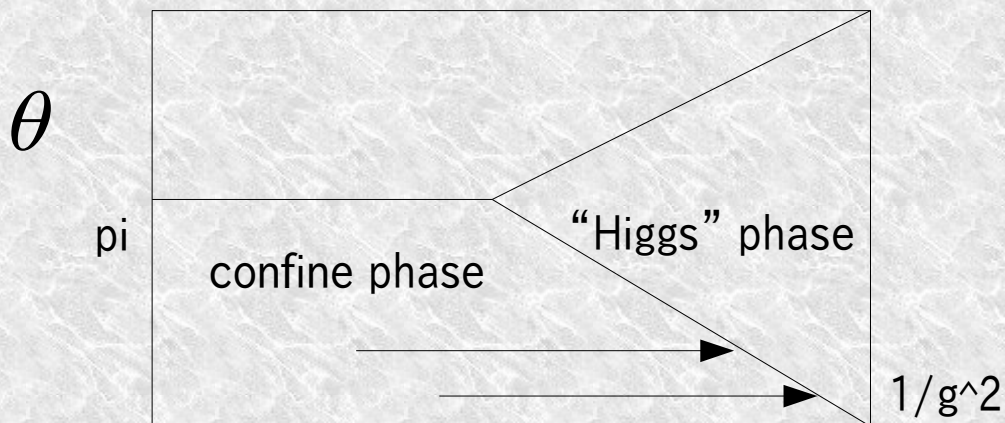


- Consistent with charge squared scaling for smaller two charges.
- Higher order effects is seen for $Q = 4$

renormalization of θ ?

- '94 Schierholz argued an interesting possibility that CP(n) & QCD may **not be confined** in the continuum limit when

$$\theta \neq 2\pi \times N$$
- '06,'07 Apenko : estimations of **RG flow** of theta in Quantum ring.
- '07 Gurosoy, Kiritsis, Nitti, in **Holographic QCD**, suggest the possibility that the **non-perturbatively renormalized theta** runs strongly into zero at IR.



Weak and Uniform E(M) field

- 98 U. Heller, 64 E. Brown construct an EM field on torus which keeps translational invariance, $U_{\square,zt} = e^{iEz}$ is constant everywhere, equiv. to a “twistier” boundary condition

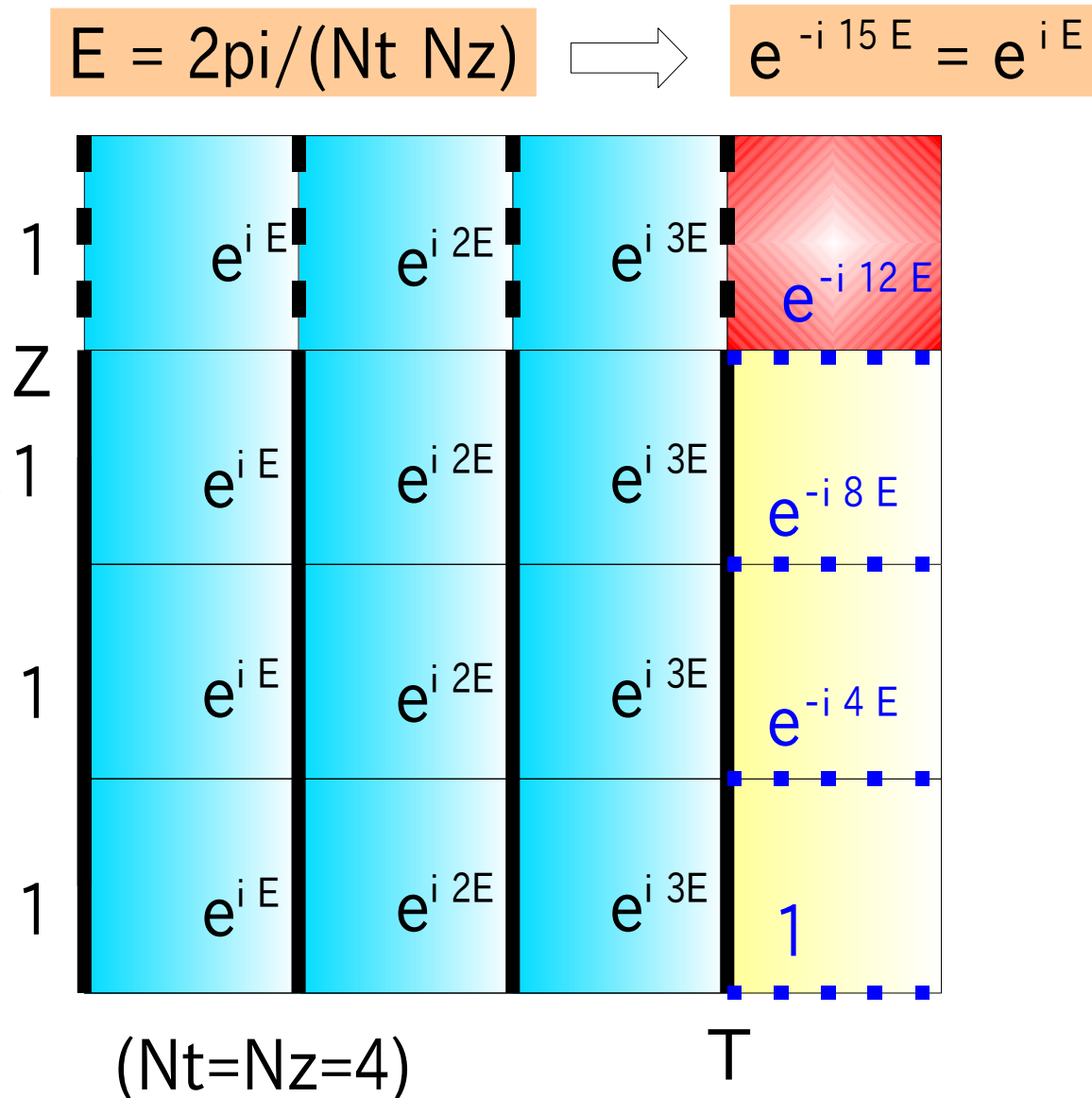
$$\psi(z, t + N_t) = \exp(iEN_x z)\psi(z, t)$$

The discretization unit is small $E_z^{(\text{unit})} = \frac{2\pi}{N_z N_t} \sim 0.01$

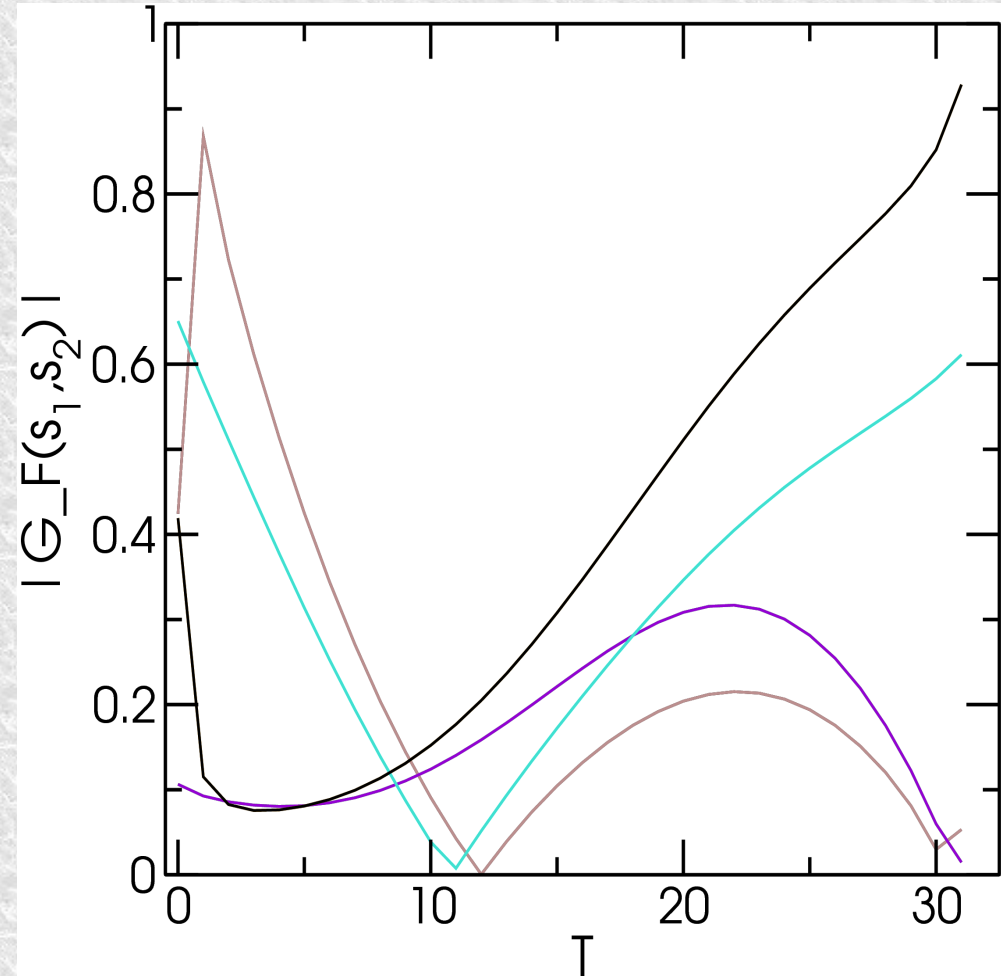
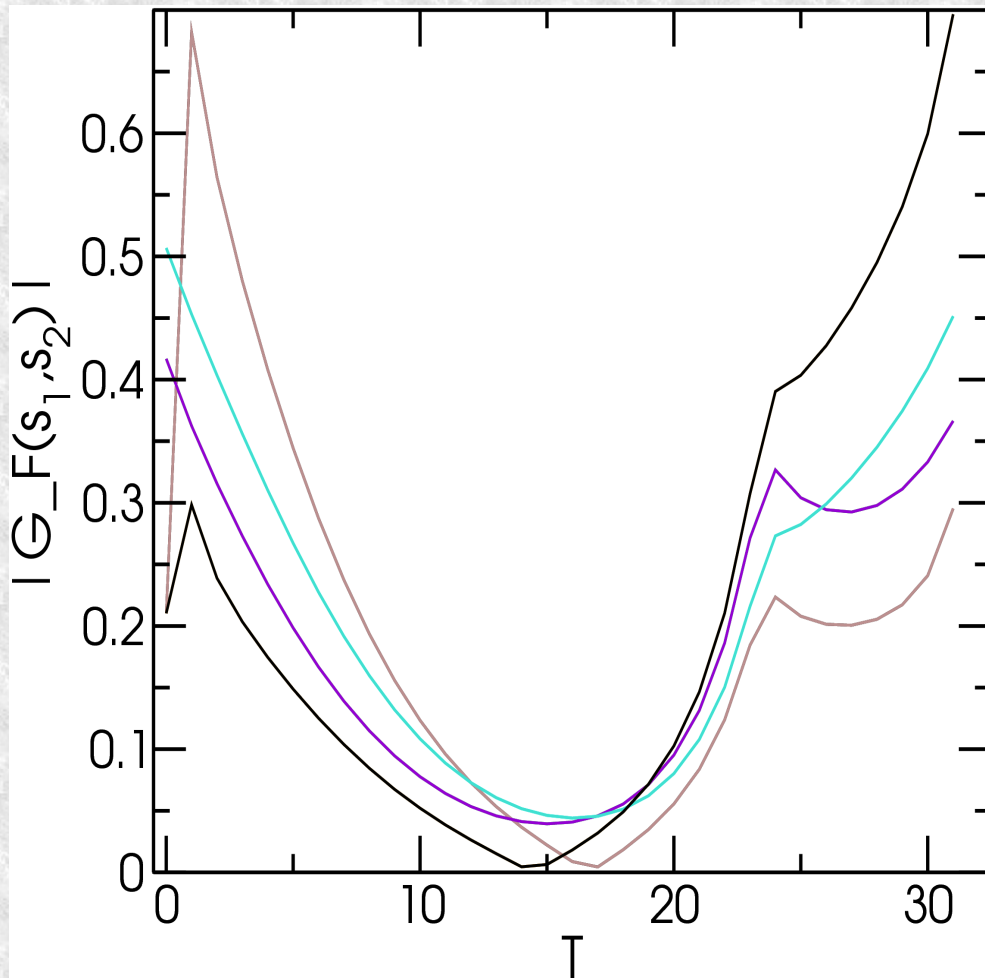
- No **strong negative field at the boundary**, which could skew the energy and/or larger excited state contamination (c.f. **delta function potential** in QM)
- Could also apply to **MDM**, **Polarizability** calcs.

Weak and Uniform external field

- Usual U(1) (unlike Minkowskian used by Aoki-Gocksch, CP-PACS) so that the energy splittings remains real number
- Weak to avoid E^γ contaminations
- Uniform to avoid a boundary effect, which causes a systematic error due to a large anti-field at the periodic boundary (05 Shintani et.al.)
- Useful for MDM, polarizability.



Free case



- $T_{src}=8$
- $T \rightarrow T-8$

- $E=E(\text{unit}) \times n$

Too much reweighting is dangerous

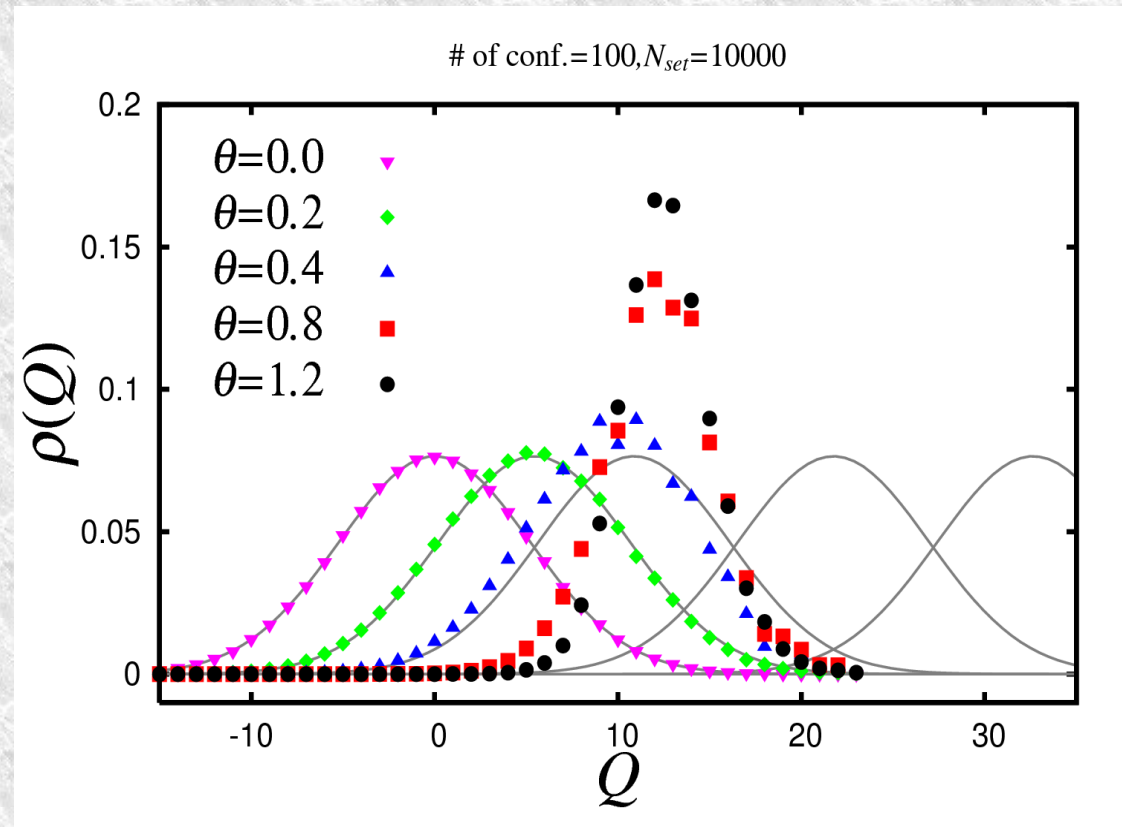
- An observable

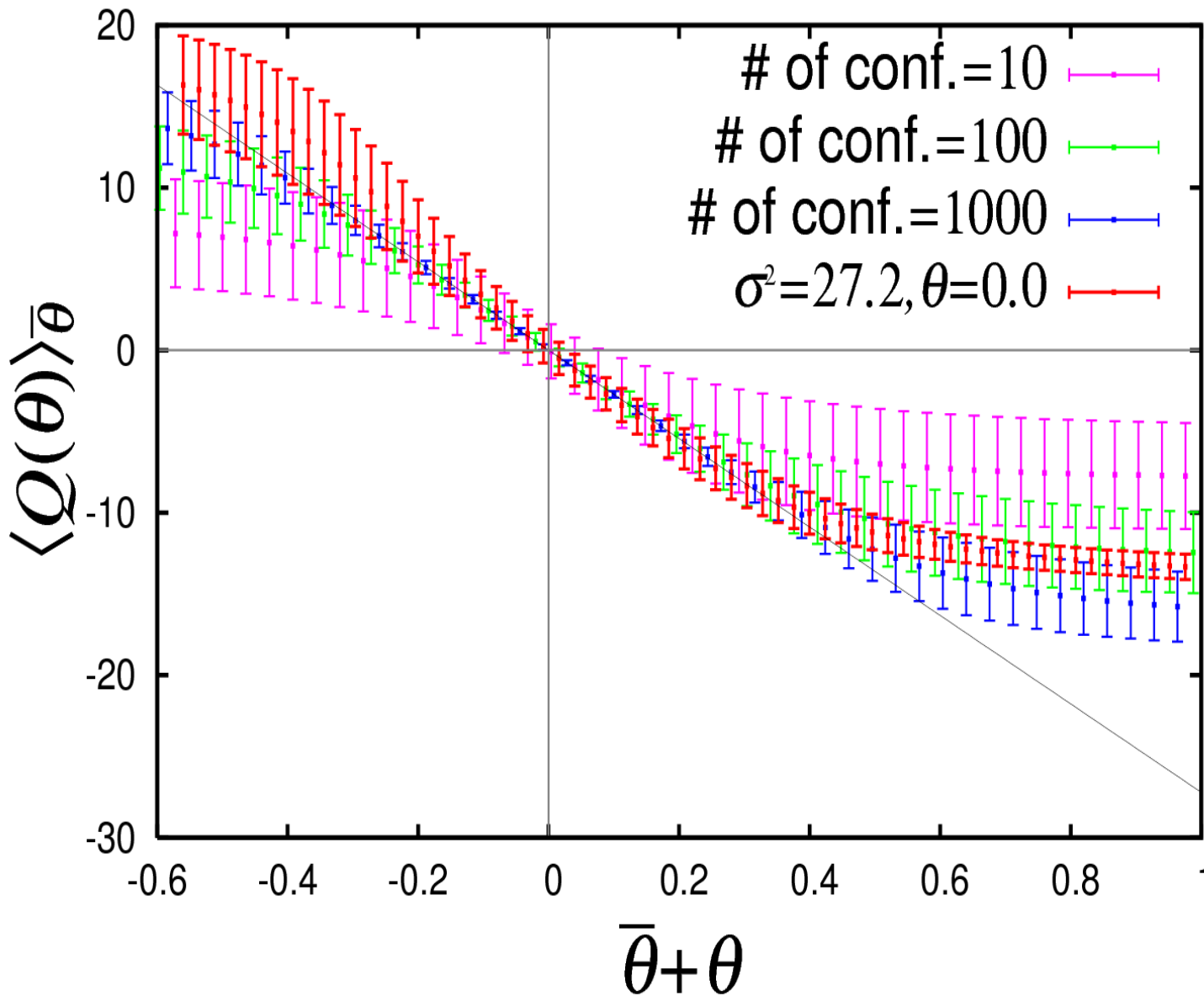
$$\langle Q e^{\theta Q} \rangle$$

always breaks the central limit theorem for large enough θ with fixed statistics.

- Gaussian mock-up data

- Nconf=100





- Large enough reweighting gives biased results and underestimate d error.
- Compared to $\theta=0.0$ real simulation data result (red), we could get a hint of # of independent configuration, ~ 100 might be appropriate.